23th May, 13 Mathematics - I

23/5/13 Sem-III (Rev) A.T.K.T.

P3-upq-Feb.-13KL-3 A4 D

Con. 5744-13. Sem III (Rev.) ATKT - Maths - I. DC-2228

(2 Hours) [Total Marks : 40

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six** questions.
(3) Use of **simple** calculator is **allowed**.

1. (a) State and verify Lagranges mean value theorems for $f(x) = x^2 - 5x + 6$ θn $[2, 4]$ **4**

OR

(a) Find the n^{th} derivative of $y = \frac{1-x}{1+x}$. **4**

(b) Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 3 & -2 & 2 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 \\ 2 & -1 & 3 & 0 & 4 \end{bmatrix}$ **4**

OR

(b) Find the length of the curve $x = \cos t + t \sin t$ and $y = \sin t - t \cos t$ from $t = 0$ to $t = 2\pi$ **4**

2. (a) By using Newton's Forward Interpolation formula estimate f (z) from : **4**

x	:	1	3	5	7
f(x)	:	2	8	14	20

(b) Find the reduction formula for $\int \tan^n x\,dx$. Hence evaluate $\int_0^{\pi/4} \tan^5 x\,dx$. **4**

3. (a) By using Adjoint method, find the inverse of $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$. **4**

(b) If $u = f\left(\frac{x^2}{y}\right)$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. **4**

[TURN OVER

4. (a) Expand $f(x) = e^{2x}$ by Maclaurins method. 4

   (b) Verify Cayley–Hamilton theorems for $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$. 4

5. (a) Verify maxima or minima for $u = x^3 - 3xy^2 - 15x^2 - 15y^2 + 72x$. 4

   (b) Find the area between the curves $y = 16 - x^2$ and $0 \leq x \leq 4$. 4

6. (a) Find the volume of a sphere of radius 'a'. 4

   (b) Evaluate $\int_0^5 (x^2+1)\,dx$ using Trapezoidal rule with $n = 10$. 4

7. (a) Attempt any **one** :— 4

   (i) Solve $\frac{dy}{dx} = (4x+y+1)^2$

   (ii) Solve $(x^3 + y^3)\,dy = (x^2y + xy^2)\,dx$

   (iii) Solve $\frac{dy}{dx} + y\tan x = \cos^3 x$

   (b) Attempt any **one** :— 4

   (i) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{3x}$

   (ii) $\frac{d^2y}{dx^2} + 9y = 4\sin x$
